Commentary

Pluto, I

1. (Move the nickel to the right of the quarter OR move the quarter to the left of the nickel.)

2. (645465 or 564546) Students might begin with what they know -- writing 2 4's with a space between them for another digit. From that point, they can *guess and check* to find the position of the other digits.

3. (12, 20) Some students will solve the problem algebraically. Others will guess and check, perhaps starting with 16 boys and 16 dogs (32 heads) which would produce 96 legs -- not enough. So some of the 16 boys will have to be dogs, to get more legs. So the guess is revised to include more dogs and fewer boys.

4. (7) Only two numbers divide into 203 evenly: 7 and 29. Since items cost more than $.10 each, there must be 7 students purchasing $.29 items.

5. (9) It takes 4 cuts to make 5 pieces and 6 cuts to make 7 pieces. If 4 cuts take 6 minutes, each cut takes 1.5 minutes. Therefore 6 cuts would take 6 x 1.5 or 9 minutes.

6. (2520) It's a combination of 7 things taken 5 at a time which translates into

$$\frac{7!}{2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$$

If students are not familiar with this combination formula, they might reason by labelling the 5 people A, B, C, D, and E, and the two empty chairs F, and G. Then there are 7 ways to fill the first chair, 6 ways to fill the second chair after the first is filled, 5 ways for the next chair, and so on down the line. Therefore there are 7 x 6 x 5 x 4 x 3 x 2 x 1 ways to fill the seven chairs, but since F and G are indistinguishable, we have counted twice as many ways as are possible. Therefore the above number must be divided by 2.

7. (1 + 1 + 1 ≠ 110) Add a line through the equal sign to make the "not equal to" sign.

8.

89 47 10 53	53 10 47 89	64 38 57 21
13 65 98 40 or	40 98 65 13 or	50 12 34 78
50 12 34 78	78 34 12 50	13 65 98 40
64 38 57 21	21 57 38 64	89 47 10 53

Also, the numbers 89 and 98 and 21 and 12 can be interchanged in any of the boxes given.

9. (786) The pattern is to triple the given number and, starting with the 2, add 2 then add 3, then 4, and so on. To find the number after 260, triple 260 to get 780, then add 6.

10.

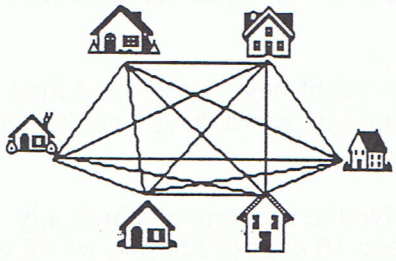
```
                2
            2       2
        2       4       2
    2       8       8       2
2       16      64      16      2
2   32  1024  1024  32  2
```

The missing term is determined by the product of the two terms located diagonally above it.

Commentary

Pluto, II

1. **(15 paths)**



2. **(6.936 or 6.94)** Add the numbers (they total 34.68) then divide that total by 5.

3. **(It's the same price either way.)** Choose any price -- $100 is easy to work with -- and work it out both ways to find it's the same. Extension: Is this true for other discount values? Why isn't the result the same as taking a 25% discount?

4. **(81 in.)** $6\frac{3}{4}$ ft.= (6 x 12 inches)+($\frac{3}{4}$ x 12 inches)=72 inches+9 inches = 81 inches.

5. **(5 twenty dollar bills or $100)** Since the total is about $93, you would need more than four twenty-dollar bills, but five is enough.

6. **(3)** One way to reason through this problem is like this. You work twice as fast as I do, so our speed working together is three times as fast as my speed alone. I.e., if my rate of work is x, then your rate is $2x$ and together we work at the rate of $3x$. Therefore if we work together, the job should take 1/3 as long as my doing the job alone. Since 1/3 of 9 hours is 3 hours, that's how long it takes if we work together.

7. **(Tuesday)** Jan. 29th is a Friday and February 1st is a Monday therefore February 23 is a Tuesday. Students might want to use a calendar to check their reasoning.

8. **(4,826,809 or 4,924,481 if you do not round)** One way to approach this problem is by working backward. If you end with 3 books after taking 23% of a number, then $0.23x = 3$ can be solved, giving $x \cong 13$. If 13 books is the square root of the previous number of books, then 13^2 or 169 is the previous number of books. If 169 is the cube root of the previous number of books, then 169^3 is the previous number of books. 169^3 is 4,826,809.

9. **(a cylinder)** If you cut and unfold a cylinder, you will have a rectangle and the top and bottom are circles.

10. **(12)** Since he jumps 5m and slides back 3m he is making 2m progress each day. The lizard's daily progress, before sliding back, can be listed at these distances in meters: 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27. It takes 12 days for the lizard to get to 27 meters, and out.

Commentary

Pluto, III

1. **(81)** $4 \Delta 5 = 4^2 + 2 \times 4 \times 5 + 5^2$
$= 16 + 40 + 25$
$= 81$

2. **(27/10)** The problem is simply a translation from 2 and 7/10 to its fractional form.

3. **(1200)** One approach is to reason that 180 jars in 15 minutes is 180÷15 jars per minute, or 12. Therefore in 100 minutes, 1200 jars are filled. Another method is to set up and solve a proportion, as shown below:

$$\frac{180 \text{ jars}}{15 \text{ min.}} = \frac{x \text{ jars}}{100 \text{ min.}}$$
$$18000 = 15x$$
$$1200 = x$$

4. **(2)** Two is the number that appears most often, and is therefore the *mode*.

5. **(There is no dirt in the hole)** All the dirt is removed from the hole.

6. **(4/52 or 1/13 or about 8%)** There are 4 such cards in the deck -- the jack, queen, king, and ace of diamonds -- out of 52 cards.

7. **(R = 9, M = 0, S = 1)** One approach to such problems is to look for numbers that must be zero or one. In this case, S must be 1 due to its far-left position in the answer. R must then be 9, and M be 0, working just with the three left-most digits in the problem.

8. **(Gold = $\frac{1}{8}$, Green = $\frac{1}{8}$, Red = $\frac{3}{8}$, Blue = $\frac{3}{8}$, Blue or Gold = $\frac{1}{2}$, Orange = 0)** The problem involves translating knowledge of fractional parts of a circle, into probability. The dartboard is visually partitioned into areas of 1/4 and 1/8 of the circle -- the answers are then obtained by putting together such regions for each color.

9. **(59)** Look at lists of the multiples of each number you're dividing by, with the remainder added. Find a common number from the lists and it will fit all the parameters. The first clue (counting by 2's, 1 left over, simply gives the list of odd numbers. The other lists are:

3's with 2 left: 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 56, 59, ..
4's with 3 left over: 7, 11, 15, 129, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, ...
5's with 4 left over: 9, 14, 19, 24, 29, 34, 39, 44, 49, 54, 59, ...

10. **(1st day = 22, 2nd day = 42, 3rd day = 62, 4th day = 82)** Students might *guess-check-revise*, by guessing what the first number might be, adding 20 to get the second, 40 for the third, and 60 for the fourth, and add to see if they get 208. If not, then revise the original guess and go through the same procedure. Another approach is algebraic, letting x be the first number, $x + 20$, the second, $x + 40$ the third, and $x + 60$ the fourth. Then $x + (x + 20) + (x + 40) + (x + 60) = 208$. This means $4x + 120 = 208$, or that $4x = 88$, so $x = 22$. Therefore the first day, the bat eats 22, the second day she eats 22 + 20, and so forth.

Commentary

Pluto, IV

1. (▮ = ▮▮▮ − ▮▮) Move one of the bars from the equal sign and place it under the subtraction sign.

2. **(a. the number of questions answered correctly in class; b. the number of minutes you have to stay after class; c. 20 minutes; d. 6)** The problem shows a real-life use of an equation. It might be interesting to ask some additional questions about this formula, such as "What happens if you answer 7 questions correctly in class, after being tardy?"

3. **(106)** The area is 900 ft^2, so $\pi r^2 = 900$ can be solved to find r = 16.9. The radius of the pen must be 16.9 ft., and the circumference of the pen is given by $2\pi r$. Therefore the farmer must buy 2 x π x 16.9 feet of fencing. When rounded to the nearest foot, this is 106 feet.

4. **(42)** Drawing a picture of the situation will help students set up an equation to solve. An equation for this situation is: $\frac{2}{3}L + \frac{1}{8}L + 10 = L$, where L is the total length of hose. Solving for l gives a total length of 48 ft., but 6 ft. of that length is taken up hanging over the boat so the length that's usable is 42 ft.

5. **(12)** There are always 12 in a dozen no matter the cost of the stamps.

6. **($0.84)** 2.35 + 1.30 + 4.99 = 8.64, and 8.64 x 1.06 = 9.16. The groceries, with tax, cost $9.16. Therefore the change from a ten -dollar bill is 84¢.

7. $(\frac{425}{999})$ Let $x = 0.\overline{425}$. Then $1000x = 425.\overline{425}$, and so $1000x - x$, or $999x$, is $4250.\overline{425} - 0.\overline{425}$, or 425. Then solving $999x = 425$ gives $x = 425/999$, as a fraction.

8. **(17)** $3^4 = 81$ and $4^3 = 64$ and 81 - 64 = 17.

9. **(a. 199 and 400; b. $2n - 1$; c. $4n$)** The pattern is established in the table. Students who are having difficulty might want to draw the next few figures, and physically count the area and perimeter of each. Tiles can be used instead of squares.

Commentary

Pluto, V

1. **(21 + 21 + 27 + 31 OR 15 + 27 + 27 + 31)** There may be more solutions.

2. **(A = 3, B = 9, C = 6)** This might be solved by *guess-check-revise*. By analyzing the problems, students know that A is less than B and C is less than B.

3. **(32)** Students might solve a proportion to find the number of yellow M&M's.

$$\frac{4}{7} = \frac{x}{56}$$
$$4x = 224$$
$$x = 32$$

4. **(hog)** When you do the computation on a calculator it gives 604, which spells "hog" when held upside down.

5. **(20)** Students might find the two numbers x and y by looking for numbers that add to twelve, and testing to see if their difference is 8. Two such numbers are 10 and 2, whose product is 20. Another approach is to solve a system of two equations in two unknowns, as shown below.

Given: $x + y = 12$
$x - y = 8$

Add the two equations to get $2x = 20$. Then $x = 10$ and $y = 2$ and $xy = 20$

6. **($16,700)** Since the dealer's cost is multiplied by 1.08 to get the final price, students can take the final price and divide by 1.08 to find the dealer's cost.

7. **($393.75)** Jason makes $200, then $100, then $50, then $25, then $12.50 and finally $6.25. The sum of these numbers is $393.75.

8. **(35)** 4 sides has 2 diagonals, 5 sides has 2 + 3 diagonals, 6 sides has 5 + 4 diagonals, 7 sides has 9+5 diagonals and so on. Another way to approach the problem is to place 10 dots spaced around a large circle, and draw in all the diagonals. Note that dots that are "neighbors" will not have a diagonal connecting them.

9. **(NEWS)** The arrows represent map directions South, East and West. The beginning letter of each direction is a letter of the word.

10. **(-13, -8, -2, 1, 6, 15)** Give students credit if their answers are within one whole number of the given answers. They might or might not have "+" in front of the positive numbers.

Commentary

Pluto, VI

1. **(34)** There is only one such time between 11:00 and 12:00, 1:00 and 2:00, 3:00 and 4:00, and 4:00 and 5:00. There are 15 such times between 12:00 and 1:00, and between 2:00 and 3:00.

2. **(12:59 p.m.)** The temptation on the part of students is to associate 1/2 with the hour factor, rather than with the minutes. If the basket was half-full at 12:59, and doubled the next minute, it would be full at exactly 1:00.

3. **(Bev = 36, Debbie = 40, Jen = 23)** Students might begin this problem by listing, using initials, the three women in their possible orders of ages -- e.g., {B, D, J}, (B, J, D}, {D, B, J}, {D, J, B}, {J, B, D} and {J, D, B}. Then the lists are eliminated one-by-one, using the clues.

4. **($\frac{1}{1000}$)** $P(8) \times P(8) \times P(8) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$

5. **($\frac{5}{24}$)** $\frac{1}{3} - \frac{1}{8} = \frac{8}{24} - \frac{3}{24} = \frac{5}{24}$

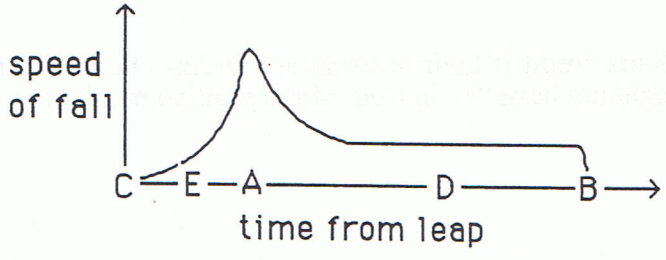
6. **(b. The area will be 9 times as much as before.)** The original area is 26 x 18 or 468 m^2. The sides become 78 m by 54 m, giving an area of 4212 m^2. The second pen is nine times the area of the first pen.

7. **(21)** \$7.00 ÷ \$0.32 = 21.875. Round down because you can not buy part of a stamp.

8. **(125 serves)** This can be solved by using a proportion. $\frac{5}{3} = \frac{x}{75}$, so $x = (5)(75) \div 3$

9. **(About 11 hours or 10.7 hours)** Answers may vary according to calculator used or rounding procedures. Here's one approach: Circumference of wheel = 83π = 260.8 in. = 21.7 ft.; 266 miles = 1,404,480 ft. The number of wheel rotations = 1404480 ÷ 21.7 = 64722.6 rotations; the number of pedal turns = 64722.6 ÷ 8.4 = 7705.1 turns; the total seconds = 5 x 7705.1 = 38525.5; the number of hours = 38525.2 ÷ 3600 = 10.7.

10. **(see below)** The positions of C, A, and B should be precisely located as shown. E can be anywhere between C and A, and D can be anywhere between A and B.



Commentary

Pluto, VII

1. **(10 stools, 32 chairs)** This problem can be approached algebraically by solving a system of two equations in two unknowns. For example, by letting S be the number of stools and C the number of chairs, we could solve $3S + 4C = 158$ and $S + C = 42$. Another approach involves simple arithmetic. Since there are at least 3 legs on each of the 42 seats, there are at least (3)(42) or 126 legs, if all were stools. The extra legs -- 158 - 126 or 32 -- can be used to make the fourth leg on 32 objects. Therefore there must be 32 chairs and 42 - 32 or ten stools.

2. **(59%)** The team won 16 times, out of 16 + 11 = 27 games.

3. **(252)** Drawing a linear diagram will help students solve this problem. 177 - 51 miles is 126 miles further to Tampa, from the half-way point. Therefore the whole distance was twice 126 or 252 miles.

4. **(4)** Diameter of circle J = 32 inches, radius of circle J = 32 ÷ 2 = 16 inches. Diameter of circle K = 16 inches, radius of circle K = 8 inches. Diameter of circle L = 8 inches, radius of circle L = 4 inches.

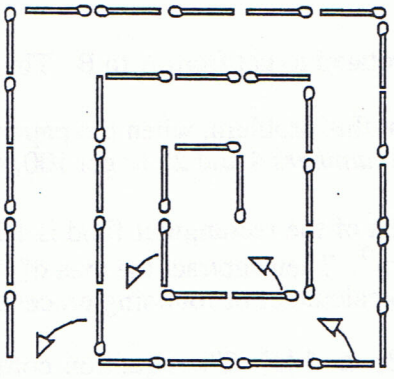
5. **(372)** Of every 13 cards, 12 are not pitchers. (403 ÷ 13) x 12 gives the number of non-pitchers in the collection.

6. **(45)** If 15 desks fit in the first 3 rows, each row has 5 seats. 5 seats per row x 9 rows = 45 total desks.

7. **(27)** This might be solved algebraically, or by *guess-check-revise*. To use *guess-check-revise*, simply start guessing the first number of miles, and add 3 for each succeeding day of the week. Add the total and see if you get 126. If not, revise the guess. An algebraic approach might start by letting x be the number of miles for the first day, then the equation $x + (x + 3) + (x + 6) + (x + 9) + (x + 12) + (x + 15) + (x + 18) = 126$ can be solved.

8. **(36/100 or .36 or 36%)** The problem involves independent probability events, which means that the probability of the two events are multiplied. The chance of hitting two free throws in a row is given by (6/10)(6/10) or (0.6)(0.6) or 36%.

9. **(See one solution below.)**



Commentary

Pluto, IX

1. **(8 sour apple)** The ratio 1 to 2 is the same as the ratio 8 to 16. The easiest way to reach that ratio is to add 8 sour apple pops.

2. **(starting at the upper left and going clockwise, the graphs are B, D, A, and C)** Students might start with a familiar situation, such as going to a movie, and discuss what happens to the crowd inside the theater. It goes up and down, about every 2 hours, so its graph would do likewise. At a motel, most people would begin leaving about 6:00 AM, so the number of people at the motel would decrease from 6:00 AM till about noon, and then gradually increase throughout the afternoon. A shopping mall would have a fairly constant crowd from about 10:00 AM through till 10:00 PM. A football stadium (on a game day) might have people come in about 6:00 at night, and stay till 9:00 or 10:00.

3. **(about 17%)** The total for the C.D.'s is $30, and so you must pay $5. This amount is about 17% of the total.

4. **(45)** Students might put 10 dots around a circle, and connect each pair of dots with a line, but only one line for each pair. If you count the lines from one point, and move around the circle clockwise, you would find these total numbers of lines to connect: 9+8+7+6+5+4+3+2+1.

5. **(a. 5; b. 50; c. 110)** There are a number of ways to compute each of these patterns of numbers. Students should be encouraged to place together compatible numbers that give "easy sums" to work with, rather than simply moving straight into the computation.

6. **(883)** Substituting 43 for m and 27 for n gives $15 \times 43 + 12 \times 27 - 2 \times 43$.

7. **(artists - 6 guilders, masons - 2 guilders)** One approach is to write the expressions $5a + 3m = 36$ and $3a + 5m = 28$ to express the information, using a as the number of artists and m as the number of masons. If you multiply the first expression by 5 and the second by 3, you get $25a + 15m = 180$ and $9a + 15m = 84$. If you subtract the second expression from the first, you have $16a = 96$, and then $a = 6$. Substituting this value into one of the two original expressions and solving for m reveals that $m = 2$.

8. **($34.15)** $42.95 x 75% gives $32.21, and this amount times 1.06 gives the cost plus the tax. **Note: always round sales tax up.**

9. **(3)** $9^3 = 729$, $\sqrt{9} = 3$. Students will likely have to guess-check-revise to find that $9^3 = 729$, but this should be relatively quick using a calculator.

10. **(1)** There are 18 units from -5 to 13, and one third of 18 is 6. Counting up 6 units from -5 leaves you at 1 on a number line.

Commentary

Pluto, XII

1. **(12)** Students might solve the proportion $\frac{10}{5} = \frac{24}{x}$. Some will reason that Caitlin's shadow is twice her height; the statue's shadow must also be twice the height - (1/2 (24) =12.

2. **(11th stroke)** Make a chart showing the strokes for each fish remembering to take into the account the different starting distance. The shark's 2 strokes to the grouper's 3 strokes, distance wise, would be the same as the shark making 14 grouper strokes to the grouper's 3, due to the shark's stroke covering 7 times the ground.

shark strokes	start	2	4	6	8	10	11
shark	0	14	28	42	56	70	77
grouper	60	63	66	69	72	75	76.5

3. **(120 g)** Visualize a golf ball balanced by 1/2 a golf ball, and a 40-gram weight. The 40 grams is the weight of 1/2 a golf ball, and 1.5 x 80 grams is the weight of a golf ball and a half.

4. **(3/9 or 1/3 or 33%)** There are 3 pencils out of 9 objects, so the chances are 3 out of 9 of drawing a pencil.

5. **(420)** The area of the 4 walls is 2 x 9 x 10 + 2 x 9 x 15, or 450 sq feet. The areas of the door (24) and window (6) are then removed.

6. **(9.75 m wide by 0.91 m high)** Change feet to inches, then inches to cm, then cm to m.

7. **(18)** One way to begin the problem is to find combinations of numbers that add to 25, and *guess and check* to see if you multiply one number by 25¢ and another by 10¢, and subtract, you get $3.80. If not, try another pair of numbers that sum to 25. An algebraic approach is to let *c* stand for the number of correct answers, and solve the equation $0.25c - 0.10(25 - c) = \$3.80$. In either approach, you should get 18 as the number of correct answers.

8. **(60)** 300 bolts in 6 minutes means the robot is working at the rate of 50 bolts per minute. The robot can therefore complete one item per minute, or 60 items in an hour.

9. **(a. odd; b. even)** The last two pages of a book must be an odd number and an even number. The sum of an odd and even number is always odd, and their product is always even. Students might want to take a few books, and try this out using a calculator.

10. **(6)** 36 pills x 4 hours per pill is a total of 144 hours. This number is divided by 24 to get 6, the number of days the pills will last.

Commentary

Pluto, XIII

1. **(125)** Students can reason that 1000 slices are needed, and 8 slices per pizza means that 1000 ÷ 8 gives the number of pizzas required.

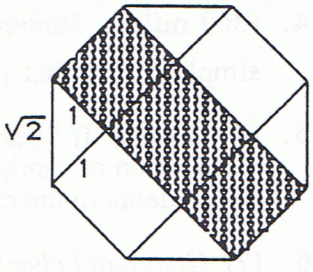
2. **(alphabetically, or during a countdown for a missile launch, or ....)** The problem is to have students think of unusual interpretations of words. Accept any answer in which four would logically follow five.

3. **($2727.27)** An equation such as $x + 10\%x = 3000$ can be used to find the beginning salary x. This equation becomes $1.1x = 3000$, or $x = 3000 \div 1.1$.

4. **(160)** Solving a proportion such as $5/2 = x/28$ will show that Ricardo has read 70 pages to Warrick's 28. Therefore Ricardo has 230 - 70 or 160 pages left to read.

5. **(Friday at 1:00 p.m.)** Add the trip time plus the layovers to get a total travel time of 21 hours and 15 minutes. Then add that total to the time of departure.

6. **(1/2)** This relationship is perhaps best seen by sketching 2 other diagonals as shown, and noticing that the small triangles are 45-45-90 triangles. By using Pythagorean Theorem $a^2 + b^2 = c^2$ and assigning 1 to length of one side of the triangle (not the hypotenuse), then as a = b, $1^2 + 1^2 = c^2$, $2 = c^2$, $c = \sqrt{2}$. The area of the shaded rectangle is: $\sqrt{2}\ (1 + 1 + \sqrt{2}\ ) = \sqrt{2}\ (2 + \sqrt{2}\ ) = 2\sqrt{2}\ + 2$. The area of the unshaded rectangles and triangles is: 4 (1/2 Base x Height) + 2 (Width x Length) = 4 (1/2) (1) (1) + 2 (1)($\sqrt{2}$)= $2 + 2\sqrt{2}$. Therefore the shaded and unshaded areas are equal. The shaded area is 1/2 of the whole.



7. **(14)** The volume of dirt taken away is 800 x 6 x 5.5, or 26,400 cubic feet. This amount, divided by 2000, gives 13.2 truckloads of dirt. Give students credit for 13 loads, 13.2 loads, or 14 loads, as removing dirt from a ditch is an approximate science, at best.

8. **(20,000)** Give students this problem when they turn in their papers:

$$(40 + 85 + 60 + 15) \times (5 + 19 \times 5)$$

9. **(b, 66)** The second drawing best describes the situation. Lionel was asleep at the midpoint of 200 miles which is at 100 miles to go; he still has to travel 1/2 as far as sleeping which would be about 66 miles asleep and 33 miles awake.

Commentary

Pluto, XIV

1. **(about 62.8 inches)** Circumference = $\pi d = 20\pi$, and π @ 3.14. Circumference = 20 x 3.14 = 62.8 inches

2. **($\frac{1}{221}$ or .005 or .5%)** The chance of pulling the first king is $\frac{4}{52}$ or $\frac{1}{13}$ and the probability of pulling the second king, given he pulled the first, is $= \frac{3}{51}$. The chances of pulling two kings in a row, then, is $\frac{1}{13} \times \frac{3}{51} = \frac{1}{221} = .005 = 0.5\%$

3. **(8, 12, 6, and 1, respectively)** Students who have trouble visualizing the cube with its painted faces might want to take 27 sugar cubes or similar blocks, and stack them up and go through the problem in a concrete fashion. The eight corner cubes will have 3 faces painted, the cubes in the middle of each of the 12 edges will have 2 faces painted, the cubes in the center of each of the 6 faces will have 1 face painted, and the cube in the very center of the large cube will be unpainted.

4. **(300 miles)** Students might solve a proportion such as $\frac{1 \text{ inch}}{40 \text{ miles}} = \frac{7.5 \text{ inches}}{x \text{ miles}}$. Or, they might simply reason that 1 inch = 40 miles, so 7.5 inches is (7.5)(40) or 200 miles.

5. **(24 OR 48)** If Sherwood got the next-to-last piece, he got piece number 49. This means that the distribution of candy started over again after 48 students. Therefore there were either 48, or half of 48, students in the class.

6. **(7)** *Guess and check* is one approach. Students might list the numbers from 2 to 9, and try different numbers until they find one that works with the other clues. Seven works, as the remainder of 45, 192, and 353 upon division by 7, is 3.

7. **(800)** Let x = season tickets, $4x$ = single admission tickets, and solve the equation $4000 = 4x + x$, or $4000 = 5x$.

8. **(36 + 16+ 1 or 49 + 4 or 36 + 9 + 4 + 4)** Other answers may be possible.

9. **(DEEDDDED)** The combination shown produces this sequence of numbers: 458, 916, 91, 9, 18, 36, 72, 7, 14. There are other combinations which will also work. An interesting extension is to find the combination with the fewest number of steps.

Commentary

Pluto, XV

1. **(1/720 or 0.1%)** The total number of ways the bookbags could be handed out is given by 6 x 5 x 4 x 3 x 2 x 1 or 720. Only one of those is correct.

2. **($25)** $800 x 0.15 = $120. Therefore she received $120 - $95 or $25 less than expected.

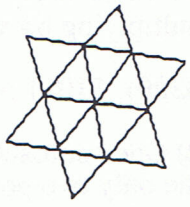
3. **(a. 87.6; b.96)** 86 + 97 +94 + 73 + 88 = 438, and 438 ÷ 5 = 87.6, her average of the chapter tests. Since the final exam counts as two chapter tests, in the end she will have a total of 7 test grades. To average 90, she must have a total of 90 x 7 or 630 points. She already has 438, so she must obtain 192 more on the final exam, counting it as two tests. Therefore the grade on the final must be 96.

4. **(7)** From the first clue, you know there is one less son than there are daughters. You can therefore *guess-check-revise* to see which combination of this nature also has the same number of daughters as twice the number of sons, less one. Three boys and four girls is the correct combination.

5. **(7776; 117,649)** The pattern is: 1^0, 2^1, 3^2, 4^3, 5^4, 6^5, 7^6, ..., x^{x+1}

6. **(77 1/4 or 77.25)** 6 x 4 is 24, 5 x 12 is 60, and 81/12 is 6 3/4. Adding the first two, and subtracting the third term, gives 84 - 6 3/4 or 77 1/4.

7. **(4/3 or 4 to 3)** The star has 12 smaller equilateral triangles formed. These 12 make up the area of the entire star and there are only 9 of these triangles in one of the big triangles.



8. **(216)** The box can hold 72 dice on the bottom layer, since 18 ÷ 3 is 6, 36 ÷ 3 is 12, and therefore the length and width of the bottom can hold 6 rows of 12 dice per row. There can be only 2 additional layers added to the bottom one, because 4 layers would be 12 cm high, which is too high for the lid to fit. The 3 layers with 72 dice per layer comes to 216 dice.

9. **(10)** The diagrams below show where the joggers are when the train first gets to the bridge, and when it gets across the bridge. The men have both run 2/5 of the way across the bridge in the first picture, so the first jogger barely escapes. The other jogger keeps going, as does the train. This jogger travels the last 1/5 of the bridge while the train is travelling the whole length of the bridge, at 50 miles per hour. The jogger must be travelling 1/5 as fast as the train, to cover 1/5 the bridge while the train is covering the whole bridge. Therefore the jogger is going 1/5 of 50 miles per hour, or 10 miles per hour.

Commentary

Pluto, XVI

1. **Plane - Nephew - Orlando** **Car - Grandparents - Tampa**
   Bus - Brother - Lake City **Motorcycle - Cousin - Miami**
   Train - Uncle - St Augustine
   Suggest that students use a chart and mark off what is known and what is impossible. By the process of elimination, they should eventually solve the logic puzzle in this fashion.

	Cousin	grands	uncle	nephew	brother	car	bus	plane	train	cycle
Orlando										
L. City										
St. A										
Tampa										
Miami										
car										
plane										
bus										
train										
cycle										

2. **($1.96)** $1.12 + $0.56 + $0.28 = $1.96

3. **(28/64 or 7/16)** Students might reason that 0.443 inches is x/64 inches, and solve for x by multiplying 64 x 0.443, arriving at x= 28.352. Rounding off, this fraction becomes 28/64.

4. **($200)** $1011 is close to $1000, and 20% of $1000 is $200.

5. **(2)** One approach is to make a chart, and distribute 5 darts in such a way as to get 120 total points. The only two possible ways to do this are shown below.

Points ›	5	10	20	50
darts ›	2	1	0	2
darts ›	0	1	3	1

6. **(6)** The Least Common Multiple of 1, 2, and 3 is 6. Another approach is to make a list:

	Mon.	Mon.	Mon.	Mon.	Mon.	Mon.	Mon.	Mon.
spelling	√	√	√	√	√	√	√	√
problems	√		√		√		√	
history	√			√			√	

   The 6th Monday after the first Monday is the next time that all three check marks appear again.

7. **(0.125 or 12% or 13% or 1/8)** The chances of drawing the first card with the letter of a month on it is 3/6 or 1/2, since A, D, and F are such cards out of the six. The chances of drawing two such in a row is then 1/2 x 1/2, and the chances of drawing three such in a row is 1/2 x 1/2 x 1/2.

Commentary

Pluto, XVII

1. **(23, 29)** Look at pairs of primes so that when multiplied, you get the desired product. One way to begin is to find the square root of 667 on a calculator, and notice that it is approximately 26. The prime numbers you are searching for will also be close to 26.

2. **(145 white; 176 teal, 120 peach; $1100 estimated cost)** Students might take a sheet of grid paper, and actually color in squares to match the tile pattern. A nice numerical pattern emerges in that the number of tiles in the borders are successive multiples of eight -- 1, 8, 16, 24, 32, 40, 48, 56, 64, 72, and 80. The white tiles are these numbers: 1+ 24 + 48 + 72 or 145. The teal tiles are 8 + 32 + 56 + 80 or 176. The peach tiles are 16 + 40 + 64 or 120. There are 145 + 176 + 120 or 441 tiles required in all. At $2.25 each, this totals $992.25 for the tiles that appear on the floor. An extra 10% must be purchased, however, and ($992.25)(1.10) gives $1091.48, or $1100 when rounded to the nearest hundred dollars.

3. **(the student's name)**

4. **(3 to 2, or, the same ratio)** Use the original ratio to determine how many boys and girls are there to start with (180 boys and 120 girls). Then figure out how many of each is left (150 boys and 100 girls) by subtraction, and write the ratio. 150:100 is the same as the original ration 6:4, or 3:2, or equivalent ways to express this ratio.

5. **(152)** $6y^2-3x + 5z$ when $x = -2$, $y = 4$, and $z = 10$, becomes (6)(16) - (3)(-2) + (5)(10), or 96 + 6 +50, or 152.

6. **(78/100 or 39/50 or 0.78 or 78%)** There are 22 muffins, out of 100, that the customer does not want. Therefore there are 78 out of 100 that would be fine.

7. **(34 inches by 52 inches)** The width of the poster and mat will be 2 1/2 feet plus 4 inches, or 34 inches. The height will be 4 feet plus 4 inches, or 52 inches.

8. **(86)** If 5 tests average 88 points, the total points of those 5 tests is (88)(5) or 440. The four scores she knows sum to 354 of those 440, leaving 440 - 354 or 86 points that the last test must be worth.

9. **(a. even; b. even; c. odd; $p = R - L - 1$)** Students might not realize, unless they take a magazine and examine it, that the first page always begins on the right, so all of the right-hand pages are odd numbers. All of the left-hand pages are then even numbers. When sheets of paper are torn out, both sides would be numbered, so an even number of pages would have been removed. The last numbered page, before the torn-out section, would have to be a left-hand page and hence an even number. R would have to be an odd number then. Students could determine an equation by listing some pages, such as: 1, 2, *3, 4, 5, 6, 7, 8,* 9, 10, ..., where the italics represent torn pages, in a concrete fashion, and generalizing to the variables used.

10. **(6)** Look at all the factors of 48 and choose only the options that fit the requirements. Those that do are: 3 by 16, 4 by 12, 6 by 8, 8 by 6, 12 by 4, and 16 by 3.

Commentary

Pluto, XVIII

1. **0 = (4 - 4) ÷ (4 + 4)**
   1 = (4 + 4) ÷ (4 + 4)
   2 = (4 ÷ 4) + (4 ÷ 4)
   3 = (4 + 4 + 4) ÷ 4
   4 = 4 + [(4 - 4) ÷ 4]

   The solutions shown to the left are only a few of the ways to do each problem. Students' work will have to be checked individually.

2. **($6.28)** (2/3) x $9.42 = $6.28. Some students might find 2/3 of $9, or $6, and then 2/3 of 42¢, or 28¢, and add those together.

3. **(12)** Make a chart showing the possibilities. One such chart is shown below:

P	25	20	15	15	10	10	5	5	5	0	0	0
N	0	1	2	0	1	3	4	2	0	5	3	1
D	0	0	0	1	1	0	0	1	2	0	1	2

4. **(a. 730; b. 729; c. 728.6)** The problem is a simple rounding situation.

5. **(27)** Students should be encouraged to begin such a problem in an organized manner. For example, they might first count all the small triangles, and get 16. Then move to the next larger size triangle -- 4 small triangles -- and find seven. There are 3 triangles the size of 9 small triangles and 1 large triangle the size of all 16 small triangles.

6. **(4/9)** Let x = 0.4444...., and then $10x$ = 4.4444.... and also $10x$ -x gives 4.4444.... - 0.4444...., or $9x = 4$. Therefore $x = 4/9$. The solution to this type of problem lies in realizing that the decimal point in numbers with repeating decimals can be "adjusted" by multiplying by a power of ten, so that subtraction of one such number from another leaves only whole numbers to the left of the decimal point.

7. **(It's a toss up.)** The average height of both teams is 5'10". Students don't actually have to find the average height -- they can simply find the total height of each team's starting five, and compare those numbers. Some students might compare the two teams by comparing individual players' heights, keeping a running total of how much taller players are when compared individually. I.e, they might find that one team is taller than the other without actually adding up the heights of all players.

8. **(Too many)** *Six dozen dozen* is 6 x 12 x 12 eggs, and *a half dozen dozen* is (1/2) x 12 x 12.

9. **(a. 738; b. 485)** Students can substitute 6 into the equation for part (a), and compute 600 + (23 x 6) to get 738. They can then substitute -5 into the equation for part (b), and get 600 + (23 x -5) to get 485.

Commentary

Pluto, XIX

1. **(2)** Computing 1,000,000 ÷ 60 ÷ 24 ÷ 365 changes minutes into years, and is 1.91 or about 2 years.

2. **((-2, -2) is incorrect; it should be (-1, -1))** The picture should be a star, symmetrical about the *y*-axis. Making this correction will result in such a picture.

3. **(Luke)** Make a list of each person and use the clues to arrange them by height. Chad would be the tallest, followed by Luke, Missy, and Mary. Therefore Luke would be 5'9", the second tallest height given.

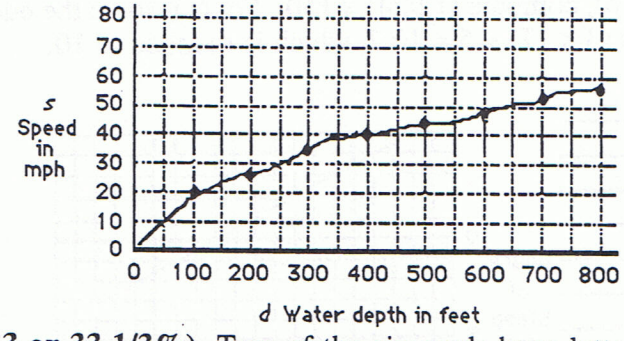
4. **(Tangent)** Get it – tan gent

5. **(95; 67)** This is a real world application of mathematics. To find the man's speed, use a calculator to find $\sqrt{360}$ as 18.97 or 19, then multiply by 5 to get 95. To find the woman's speed, find $\sqrt{180}$ as 13.4 and multiply by 5 to get 67.

6. **(14 mL, 55 mL)** Divide the weight by 6 and multiply by 3 mL. Or, solve the proportions $3/6 = x/28$ and $3/6 = x/110$.

7. **(5 " by 5", 80 tiles)** One way to find the size of the new and old tiles is to look at the areas of tiles which are squares, and find two areas that differ by 39. The first few square numbers are 1, 4, 9, 16, 25, 49, 64, 81, 100, ..., and notice that 25 and 64 differ by 39. So the old and new squares are 5-by-5 and 8-by-8 inches, respectively. Then divide 5120 by 64 to get 80.

8. **(See below.)** Students should again use a calculator to find the square root of the numbers 100, 200, 300, 400, 500, 600, 700, and 800, and multiply each resulting value by 2. This produces the points (100, 20), (200, 28), (300, 35), (400, 40), (500, 45), (600, 49), (700, 53), and (800, 57) to graph, when the *y* values are rounded to the nearest whole number.



9. **(2/6 or 1/3 or 0.33 or 33 1/3%)** Two of the six cards have letters on them that could represent days of the week -- S and T. Then the chances are 2 out of 6 that you will draw one of these cards.

Commentary

Pluto, XX

1. **6 = [(4+4) ÷ 4] + 4**
   7 = (4+4) - (4÷4)
   8 = (4+4) + (4–4)
   9 = (4+4) + (4÷4)
   These are just a few of the many ways of writing these numbers. Each student answer different from these should be checked individually. If no grouping symbols are used, the order of operations rule is: do multiplication and division first (in order from left to right in the equation) and next do addition and subtraction (in order from left to right in the equation).

2. **(14 or 15)** Make a chart for hiccups and minutes accumulated. Then use the information that April has 30 days or 30 days x 24 hours/day x 60 minutes/hour = 43,200 minutes. The minutes pass in a "powers of two" pattern -- 1, 2, 4, 8, 16, ..., or 2^0, 2^1, 2^2, 2^3, 2^4, ... and the total number of minutes accumulated per hiccup follows a "one less than the powers of two" pattern: 2^0-1, 2^1-1, 2^2-1, 2^3-1, 2^4-1, ...The power of two closest to 43,200 is 2^{15} or the 32,768th minute into April. Depending on when the hiccup occurs during that span, either 14 or 15 hiccups is an acceptable answer.

3. **(24)** Students might either list all of the arrangements using the four names, or realize that there are 4 ways for the first position to be occupied, and once that's done, 3 ways for the next, two ways for the next, and only one for the last. Therefore there are 4 x 3 x 2 x 1 ways for them to arrange themselves.

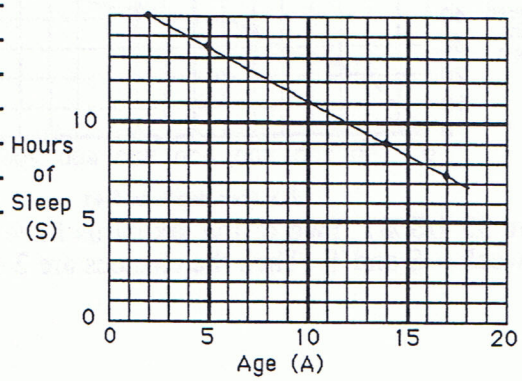
4. **(8 races, 36 races)** Make a list for the number of wins for each race to keep track of them all. By the 8th additional race, she will have won 14 times out of 28 races, which is 50%. By the 36th additional race, she will have won 42 times out of 56 races, which is 75%.

5. **(14)** Students might make a Venn Diagram by starting with the 5 players who play both positions. Then there must be 6 who only pitch, and 1 who only plays first base. Therefore there are 26 - 5 - 6 - 1 players who neither pitch nor play 1st base.

6. **(a. 91 mg; b. 75 mg.; c. 10)** For (a), compute (10/22) x 200 and round to the nearest whole number. For (b), compute (12/24) x 150. For (c), solve the equation for *A* by using 3 = (5/17)*A*. Then *A* is (3 x 17) ÷ 5 = 10.2, which is rounded to 10.

7. **(See below.)**

A	*S*
2	15
5	13.5
10	11
14	9
17	7.5



Commentary

Pluto, XXI

1. **(4)** It is possible that the first 3 pieces pulled will all be different, but the fourth piece has to match one of the first 3.

2. **(18)** 6 sec. + 6 sec. + 6 sec. = 18 seconds

3. **(5 feet, 4 and 3/4 inches)** The problem involves subtracting 12 feet, 2 and 1/4 inches from 17 feet, 7 inches. Some students might have difficulty subtracting 2 1/4 inches from 7 inches.

4. **(January 9)** The phenomenon is that, when written in the common abbreviated form, it always states the current year: 1/9/97, 1/9/98, 1/9/99. This will not be possible again until 2/1/00 (February 1, 2100) because, of course, February has no "0" date to use in 2000, nor the rest of the century.

5. **(14)** The number of cars can be found by subtracting 8 from 32 and dividing by 4, giving 6 cars. On the trip home, 3 passengers per car means that 18 people rode in cars, leaving 32 - 18, or 14, to ride the bus.

6. **(Katie)** The three heaviest total 505 pounds, Katie being one of those. However, this is too much weight, so Katie must switch with Amie. Therefore Katie must stay on shore.

7. **(9%)** 215 -195 = 20, and 20 ÷ 215 = 0.093 or 9%.

8. **(3024)** Find the depth D by using 3 x 2 x D = 9 ft^3, so D = $1\frac{1}{2}$ feet. The measurement of the box in inches, then is 24-by-36-by-18. The 5 surfaces covered have these areas: two that are (36 x 18); two that are (24 x 18); one that is (24 x 36).

9. **(the c.d., poster and book <u>or</u> shorts, poster and book)** The problem involves simply adding the items three at a time, and seeing which have a sum less than $30.

Commentary

Pluto, XXII

1. **(20)** 12:44, 1:44, 2:44, 3:44, 4:04, 4:14, 4:24, 4:34, 4:40, 4:41, 4:42, 4:43, 4:44, 4:45, 4:46, 4:47, 4:48, 4:49, 4:54, 5:44

2. **($36,000)** You would get 1,000,000 ÷ 20 or $50,000 per year, before taxes. A tax of 28% means that you get to keep 72%, and 72% of $50,000 is $36,000.

3. **($3159)** There would be 180 x 13 years you would attend school, or 2340 days. At $1.35 per lunch, you would spend $3159, or over $3000.

4. **(a. 8 minutes, 20 seconds or 8 minutes; b. 26.4 trillion miles)** For (a) 93,000,000 ÷ 186,000 = 500 seconds, and 500 seconds is 8 minutes and 20 seconds. For (b), changing 186,000 miles per second into miles per year can be done by computing 186,000 x 60 x 60 x 24 x 365, which gives 5,865,696,000,000 miles. (Note that this computation can be done on an 8-digit calculator by leaving off the 0's, and appending them to the product of the significant digits.) Then 4.5 light years would be approximately 26,400,000,000,000 miles. Accept any answer between 25 and 28 trillion miles. In scientific notation, this would be 2.64×10^{13}.

5. **(20%)** From 70 to 56 is 14, and 14 ÷ 70 = 0.20 = 20%.

6. **(It was a fake.)** The coin has "B.C." engraved on it, but it would be impossible to put this on a coin prior to the birth of Christ.

7. **(Dorrie, Reynaldo, Evelyn, Rocky)** Students might want to try this formula themselves. The calculations are shown below for the four students listed:

   Dorrie: (5 + 7)(14 + 6) = 240
   Reynaldo: (6 + 4)(13 + 10) = 230
   Rocky: (2 + 3)(12 + 4) = 80
   Evelyn: (6 + 8)(11 + 2) = 182

8. **(a. 10; b. 15; c. 21; d. 5,151)** Students should make the next few rectangles and count the total for each vertical line, and search for a pattern. The list below shows the first few results.

no. of vertical lines:	0	1	2	3	4	5 ...., x
no. of rectangles:	1	3	6	10	15	21 ..., $\frac{[x+1][x+2]}{2}$

   If 100 is substituted for x in the pattern above, we get (101)(102) ÷ 2, or 5151.

Commentary

Pluto, XXIII

1. **(1st - 7 books; 2nd - 9 books; 3rd - 8 books)** The stacks are close to 24 ÷ 3 books high, or 8 books high. Let 8 be the middle stack, and 8 + 1 and 8 - 1 the other two.

2. **(a. 108° F; b. 3.6 kilometers)** Part (a) involves substituting 1 for k and computing $T = 68 + 40 \times 1$, or 108. In part (b), the student will have to solve $212 = 68 + 40k$ for k by subtracting 68 from both sides, leaving $144 = 40k$. Then divide both sides by 40, producing $3.6 = k$.

3. **(the 12-oz bottle)** \$1.38 ÷ 12 = \$0.115 per oz., while \$1.02 ÷ 8 = \$0.125 per oz. Since \$0.115 < \$0.125, the 12 oz. bottle costs less per oz.
Another way is to use the LCM - 24. Two 12 oz. bottles would cost \$2.76 and three 8 oz. bottles would cost \$3.06.

4. **(3.75π or 11.775 in^2)** The diameter of the disk is 4 inches, after the non-playing margin is removed. Its area is then $\pi \times r^2$ or $\pi \times 2^2$. The area of the center part which doesn't play is $\pi \times (\frac{1}{2})^2$. The difference between these is $4\pi - \frac{1}{4}\pi$ or 3.75π.

5. **(1/4 or 25%)** The probability it will *not* rain on Saturday is 50%; the probability it will not rain on Sunday is also 50%. Therefore the probability it will not rain on Saturday followed by Sunday is 50% x 50%, which is 25%. Students might want to convince themselves of this by flipping coins or through some other simulation.

6. **(a. Answers will vary; b. $L = a + 1.5y$, where a is the answer from part a; c. Answers will vary.)** The students will likely get about 1/2 inch for part (a), which means that part (b) will become $L = 0.5 + 1.5y$. To find (c), students solve the equation in (b), using 37 for L, and searching for y. In the case of $L = 0.5 + 1.5y$, solve $37 = 0.5 + 1.5y$ by subtracting 0.5 from both sides, then dividing both sides by 1.5. The answer in that case would be 24.3 years.

7. **(4.2×10^{11})** Multiplying 140,000 by 3,000,000 gives 420,000,000,000. This number is 4.2×10^{11} in scientific notation.

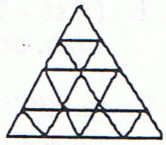
8. **(2×10^{-5})**

9. **(5/36)** There are usually six ways to have a sum of seven when two dice are thrown: (1, 6); (2, 5); (3, 4); (4, 3); (5, 2); (6,1)
One of the possibilities above would be eliminated by a blank 2 on one die and a blank 5 on the other; notice that the other 2 and 5 combination would still be possible. Therefore there are only 5 chances out of 36 of throwing a sum of seven.

Commentary

Pluto, XXIV

1. **(a. 24; b. 96)** In the far right figure on the worksheet, you can see that 4 such triangles would be made in each of the six sections, giving 24 for a side of 2 inches. The figure to the right shows that each section would have 16 triangles, if the side length was 4 inches.



2. **(a. 2.68×10^{11} or 2.7×10^{11}; b. 3.4×10^{17}; c. 1,270,000 or 1,260,000)** The earth's volume is found by computing $\frac{4}{3}\pi (4000)^3$, which is 2.68×10^{11}. The sun's volume is $\frac{4}{3}\pi (433{,}000)^3$ or 3.4×10^{17}. Dividing 3.4×10^{11} by 2.68×10^{11} gives about 1,270,000. Therefore it takes about one and a quarter million earths to fill up the sun.

3. **(9 nickels, 3 dimes, and 1 quarter; or 5 pennies, 7 dimes, and 1 quarter; or 5 pennies, 3 nickels, 3 dimes, and 2 quarters; or 6 nickels and 7 dimes)** Students might make a chart to help them decide on the different possibilities.

4. **(3,456,000)** One approach is to compute $360 \times 20 \times 60 \times 8$.

5. **(a. locker #60; b. 26 lockers, 1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 97)** a. opened every month = locker # 60 (multiples of 60); can use least Common Multiple (LCM) to find the Locker number(s) that are searched every month; LCM of 2 and 4 = 4; then the LCM of 4 and 3 = 12; then the LCM of 12 and 5 = 60.
   b. not opened lockers = prime numbers >6, 1, 49 (7x7), 77 (7x11), 91 (7x13) = 26 lockers

6. **($44.00)** If she had $11 at the end of the day, she had $22 prior to that, and $44 prior to that. You can check that this is correct by taking half of $44 to get $22, and then take half of $22 to get $11.

7. **(3/4)** Students with good number sense know that 25% off means you pay 75% of the price; they also know that 75% is 3/4.

Commentary

Pluto, XXV

1. **($321.66)** This problem is much easier if students use in their computation not the percent off for each discount, but the percent that the coach has to pay. For 25% off, the coach pays 75% of the price; for 8% off, he pays 92%; for 5% off, he pays 95%. The equation $(75\%x)(92\%)(95\%) = \$210.85$ describes the situation, where x is the original price. Then $x = \$210.85 \div (0.75 \times 0.92 \times 0.95)$ can be easily computed on a calculator.

2. **(0)** Students with good number sense will look ahead in this problem, rather than blindly charging ahead and computing from the left end. They will see the zero and know that the final answer will therefore be zero.

3. **(a. $1610.51; b. $2593.74; c. $4177.25; d. $7400.25)** Hopefully students will realize that each year's total is given by multiplying the previous year's total by 1.1, as this automatically adds on the 10% interest to the previous balance. Then the repeating function feature of most hand-held calculators will give each year's new total simply by pressing [=].

4. **($12.60; $1.80)** $54 ÷ 6 means that each of the six meals came to $9. The four non-birthday folks would then pay $9 plus their share -- 1/5 -- of the birthday girls' meals. Each birthday girl would pay her fair share, 1/5, of the other girl's meal. As 1/5 of $9 is $1.80, Harry, Pam, Andy and Beth each pay $9 + $1.80 + $1.80, or $12.60. Ann and Joan each contribute 1/5 of $9, or $1.80

5. **(4)**

Ben		1 student
at 8:00 a.m.	5 flyers	6 students
at 9:00 a.m.	25 flyers	31 students
at 10:00 a.m.	125 flyers	156 students
at 11:00 a.m.	625 flyers	781 students
at 12:00 noon	3125 flyers	all students

6. **(81)** $\frac{56}{80} = 0.7$ hours for the first leg; $\frac{60}{75} = 0.8$ hours for the second leg; $\frac{46}{92} = 0.5$ hours for the third leg. The total travel time is 2 hours, $\frac{162}{2} = 81$ km/hr

7. **(-46, -48, -50, -52, -54)** Divide 250 by 5 to find the average (middle) and use that to center the others around.

8. **(10)** The number trick is justified by this method, where x is the number of people in the house. First step gives $2x$; second step gives $2x + 5$; third step gives $20x + 50$; fourth step gives $20x$; fifth step gives 10.

9. **($220.50)** The expression that gives her earnings is $\$4.50 \times 40 + 1.5 \times \$4.50 \times 6$, or $180 + $40.50. The sum of regular pay and overtime is then $220.50

Commentary

Pluto, XXVI

1. **(a. 20; b. 35; c. 56; d. 220)** The apples in each layer follow the pattern:

   1, 3, 6, 10, 15, 21, 28, 36, ...

   You can get from one layer to the next by adding one more number than previously. Or, for the *nth* layer, $(n)(n + 1) \div 2$ gives the number of apples in that layer.

2. **(a. 199; b. $2n - 1$)** The number of fleas each day follows the pattern of odd numbers: 1, 3, 5, 9, 11, ..., $2n - 1$, ...

3. **(0.0001)** The number is first given in scientific notation. The student must know how to convert scientific notation to a decimal equivalent.

4. **(3/8)** There are 8 letters in *multiply*, and 3 of them -- u, p, and t -- also appear in *product*.

5. **(144)** There are 8 possibilities for the first digit, 2 for the second, and 9 for the third. Therefore there are 8 x 2 x 9 = 144 total ways.

6. **(15)** A right triangle is formed, with the hypotenuse being 25 feet and one leg being 20 feet. Therefore the Pythagorean theorem can be used: $a^2 + b^2 = c^2$. If we know $a = 20$, then we can solve for b in this fashion: $b^2 = 25^2 - 20^2 = 625 - 400 = 225$. So $b = \sqrt{225} = 15$.

7. **(a. $n + 5 \times -11 = 10$; b. 65)** There are other forms of the equation shown which are equivalent. Some of them are: $n + -55 = 10$; $n - 55 = 10$; $n + -11 + -11 + -11 + -11 + -11 = 55$, and so on. Each of these equations can be solved by adding 55 to both sides, resulting in $n = 65$.

Commentary

Pluto, XXVII

1. **(432 + 432 = 864)** Many solutions, check individual problems.

2. **(.5)** 512 ÷ 16 = 32. Then (32 ÷ *N*)+ 256 = 320, therefore 32 ÷*N* = 64. N = 32 ÷ 64, 0r .5

3. **(c.)** a. $14.60; b. $17.60; c. $14.30; d. $15.30

4. **(17280)** There are 5 x 4 x 3 x 2 x 1 ways to arrange the baseball trophies, 3 x 2 x 1 ways to arrange the tennis trophies, and 4 x 3 x 2 x1 ways to arrange the soccer trophies. Therefore there are 120 x 24 x 6 = 17280 ways to arrange all the trophies.

5. **(1 hour 30 min. = 90 min.)**

6. **($76.19)** $\frac{86.75 + 42.50 + 105 + 70.50}{4}$ = $76.19

7. **(24)** There are 11 campers who counted between, but not including, 5 and 17. So there must be that same number of campers between 5 and 17, but on the other half of the circle. 1, 2, 3, 4, 18, 19, 20, 21, 22, 23 and 24 would be the numbers.

8. **($41,580)** Many students will have the answer $42,000, thinking that a 10% pay cut and a 10% pay raise should offset each other. However, the base upon which the 10% is taken changes. 10% off of $42,000 means he was making $42,000 x 90%, or $37,800. Then a 10% pay raise on that amount is $37,800 x 1.1, or $41,580.